How To Do Word Problems

One of the greatest difficulties and teaching opportunities I see as a math tutor relates to how to do word problems. In this tutorial, I will explain how I approach the topic with my students.

Before beginning, the frequent question I get from my students is; "Why do I have to learn this stuff, when I will never use it?" My answer always is "Math teaches to think and think in a logical, orderly manner. Thinking can be used in every aspect of life." Perhaps, this is why politicians don't like supporting math education. It teaches people to think.

Fear Factor: What strikes fear in math students is that word problems look VERY different than problems expressed as equations. So, I explain that word problems simply add one additional step. The problem is that the new step is the first step in solving a word problem.

Translation: That first and new step is simply the translation from a human language to the language of equations. If you think about it, we routinely describe real world problems to ourselves in human language, making the skill of translation useful throughout life. After translation, it is the same math the student should have already learned. Therefore, this article is intended to teach the art of translation from human language, English in my case, to the language of mathematics, equations.

Translation & Solution Procedure

Step 1: First, read the problem in its entirety. Identify what the problem is about. <u>What is the question?</u> It could be about red versus blue marbles or money or time or distance or anything else.

Step 2: Figure out <u>what is not important</u>. A school problem and certainly real life problems can throw extraneous information at you, which could throw off the thinking process if not ignored. So, cross out unimportant information.

Step 3: Before beginning any work, <u>form a **strategy**</u> as to how you plan to approach the problem.

Step 4: Assign <u>**unique** variables</u> to things that you need, but don't know and need to determine. Until you can do this in your head, make a data **dictionary**, explaining exactly what each variable represents. For example:

R = the number of Red marbles
B = the number of Blue marbles
A = the number or amount of something Adam has or is
C = the number or amount of something Charlie has or is

Avoid letters that are not unique or can be mistaken for numbers if not written clearly. For example, "S" can be mistaken for "5" or "O" as "0" or "I" as "1".

<u>Draw pictures or diagrams</u>, if helpful or necessary.

<u>Be neat and orderly</u>, using enough space and paper as necessary. The teacher should be able to clearly follow the flow of your solution. (Most students tend to scribble and, therefore, get themselves confused.) Do the problem on scrap paper and then copy to your homework or test.

Step 5: Follow the basic rules, which usually, but not always apply:

A, Each single sentence usually represents a single equation.

B, The verb (is, was, has, etc.) in the sentence usually represents the equals sign (=).

C, Identify key words (listed below) which represent mathematical operations.

D, Decide on a strategy for solving the problem.

E, Use consistent units for all variables, changing units if necessary.
Smaller units are usually better, avoiding fractions and decimals.

F, Beware of quirks in the English language and make compensations.

G, Translate each sentence into its corresponding equation.

Step 6: Solve the problem. At this point, the word problem should have be converted into a set of equations, which you can solve in the same manner that similar equation-based problems are solved.

Step 7: Check your work.

Does the answer make sense?
Do the problem backwards, using inverse operations, to see that you get the same starting point.

Key Words

Operation	Key Words
Addition	Increased by, more than, combined together total of, sum, plus added to, any comparative, such as greater than, farther, taller, etc.
Subtraction	Decreased by, minus less difference between difference of, lessthan fewer than left, after, any comparative, such as smaller than, shorter, etc.
Multiplication	Times multiplied by, product of, increased by a factor of, decreased by a factor of, twice, triple, etc.
Division	Per out of, ratio of quotient of percent, equal pieces, split, average
Equals (verbs)	Is, are was were, will be gives, yields costs or any verb
Note: Percent means divide by 100.	

Quirks in the English language

The biggest quirk relates to **subtraction**, expressed with such words as “less than, fewer than, younger than, shorter than etc.” For example:

“Adam has 5 fewer marbles than Charlie” translates to “A = C – 5” not “A = 5 - C”

Note the reversal of terms! Subtraction (but not addition) depends on the order of the terms. Getting it wrong will result in negative numbers instead of positive numbers or vice versa. If in doubt, pick a suitable number for Charlie and plug it into the equation. If Adam appears to have a negative number of marbles, which is impossible, the mistake is obvious.

Example Word Problems

Each of the examples word problems below demonstrates some aspect of the process and all require some level of planning a strategy.

A typical sales problematic

A store sells two sizes of a product. The smaller one sells for $8.95 and the larger one sells for $15.95. One day, the store sold twice as many of the larger products as the smaller products. If the total sales for that day for these products was $694.45, how many of each size were sold?

Let: S = the number of smaller versions of the product – an answer we want
L = the number of larger versions of the product – the other answer we want

Solution:	Numbers from the first sentence:	L = 2S
	Money from the second sentence:	\$8.95 x S + \$15.95 x L = \$694.45
	Substituting:	\$8.95 x S + 2 x S x \$15.95 = \$694.45
	Combining terms:	\$40.85 x S = \$694.45
	Dividing by \$40.85:	S = **17 smaller**
	Substituting in the first equation	L = 2 x 17 = **34 larger**

Note that there are two variables and two equations. The equations are about two different aspects of the problem, one about the number of items sold and the other about the money.

A percentage proble

What percent of 80 is 32?

Let P = the percentage – the answer we want

$$P\,x\,80 = 32 \quad , \quad P = \frac{32}{80} = 0.40 = 40\ percent$$

Note that "percent" means "divide by 100." Therefore, to get a percent, we must multiply by 100.

The parallel work flow problematic

Team A can dig a trench in 8 Hrs and Team B can dig the trench in 12 Hrs. If they start together at opposite ends of the trench and don't interfere with each other, How long will it takes for the two teams to dig the trench?

Let: A = the time for Team A to dig the trench alone
B = the time for Team A to dig the trench alone
T = the time for both Teams A & B to dig the trench together – the answer we want

Solution: $$\frac{1}{T} = \frac{1}{A} + \frac{1}{B}$$

Multiplying by T, A, and B:

$$T = \frac{A\,x\,B}{A+B} = \frac{8\,x\,12}{8+12} = \frac{96}{20} = 4.8\,Hrs = 4\,Hrs\,,\,48\,Min$$

In this problem, if you do not know to use reciprocals, you will get a ridiculous answer that makes no sense.

Now, this problem can be done in a different manner.

If team A can do the job in 8 Hrs, it can do 1/8 of the job in one hour.

If team A can do the job in 12 Hrs, it can do 1/12 of the job in one hour.

Working together, teams A and B can do the following portion of the job in one hour:

$$A+B=\frac{1}{8}+\frac{1}{12}=\frac{3}{24}+\frac{2}{24}=\frac{5}{24}$$

This means that the time it will take them to do the complete the job is:

$$\frac{25}{5}=4.8\,Hrs=4\,Hrs,\,48\,Min$$

The sale and sales tax problematic

At shirt is on sale for "20% off." Sara bought the shirt and also bought a pair of socks for $2.99. After including the 7% sales tax, she paid a total of $28.87. What was the original price of the shirt and how much did she save.

Let: P = the original price – the answer we want
20% of P = 0.2P is the discount – the other answer we want
80% of P = 0.8P is what she paid for the shirt
(Note that 100% - 20% = 80% = 0.8.)

Solution:	1.07 x (0.8P + $3.00) = $28.88
Dividing by 1.07:	0.8P + $3.00 = $26.99 (pre-tax price)
Subtracting $3.00:	0.8P = $23.98
Dividing by 0.8:	P = **$29.99**
Savings:	0.2P = **$6.00** pre-tax and **$6.42** after tax

Doing things backwards problem

Most problems provide a starting point and have you compute the ending point. This problem gives you the ending point and asks you to compute the starting point, using the translation principles given above. Note that the strategy is to do the problem in the reverse order.

Joy is having a party and has a fixed amount of money to spend on preparation. She spend 1/3 of the money, plus $20 on food. With the money left over, she spends ½ of it plus $10 on drinks. With the money left over from the second purchase, she spends ½ of it plus $10 on paper plates and has $5 left to buy a coffee. How much money did she start with?

Let: X = the amount of money she started with – what we need to determine
Y = the amount after the first purchase and before the second purchase
Z = the amount after the second purchase and before the third purchase

Equations 1, 2, and 3: $X-\frac{X}{3}-\$10=Y$, $Y-\frac{Y}{2}-\$10=Z$, $Z-\frac{Z}{2}-\$10=\$5$

Can we solve the first or second equations? No, because a single equation has two variables. Can we solve them as simultaneous equations? No, because we have only two equations and three variables. Can we solve the third equation? Yes and once that is solved, the result can be substituted into the previous equation.

$Z-\frac{Z}{2}-\$10=\$5$ becomes $\frac{Z}{2}=\$15$ which becomes $Z=\$30$ which we plug in equation 2

$Y-\frac{Y}{2}-\$10=\$30$ becomes $\frac{Y}{2}=\$40$ which becomes $Y=\$80$ which we plug in equation 1

$X-\frac{X}{3}-\$20=\$80$ becomes $\frac{2X}{3}=\$100$ which becomes $X=\frac{3}{2}x\,\$100=\$150$ the answer

Dealing with dimension conversion

In this ridiculous example, I will use my favorite and antique British unit of speed, “furlongs per fortnight.” I use a technique called “dimensional analysis” where the dimensions are carried along with the values throughout the calculation. “Conversion factors” are used to change dimensions, introducing new dimensions and canceling out old dimensions without confusion. The same technique would be used to convert different dimensions of the same type into a set values of a single consistent dimension.

> A British race car was clocked at 334,120 furlongs per fortnight. What was its speed in miles per hour and kilometers per hour?

First, we need to know what a furlong and fortnight are, so as to create our conversion factors.

> 1 Furlong = 1/8 mile = 220 yards = 660 feet
> 1 Fortnight = 2 weeks = 14 days = 336 hours

Now, we create our conversion factors, knowing that we always multiply by 1 to convert in either direction.

$$1=\frac{8\,Furlongs}{1\,mile} \quad \text{or} \quad 1=\frac{1\,mile}{8\,Furlongs} \quad \text{and} \quad 1=\frac{336\,Hrs}{1\,Fortnight} \quad \text{or} \quad 1=\frac{1\,Fortnight}{336\,Hrs}$$

Using these conversion factors and canceling out the dimensions furlongs and fortnights:

$$\frac{335{,}120\,Furlongs}{Fortnight}\,X\,\frac{1\,mile}{8\,Furlongs}\,X\,\frac{1\,Fortnight}{336\,Hrs}=123.3\,\frac{miles}{hour}$$

¿

Likewise with conversion factor “1 mile = 1.609 kilometers,” canceling out the dimension miles:

$$\frac{123.3\,miles}{hour}\,X\,\frac{1.609\,kilometers}{mile}=200\,\frac{kilometers}{hour}$$

Epilogue

I hope this set of instructions and sample problems will help anyone to approach word problems without fear, devise the correct strategies and obtain the correct results.

One of my favorite YouTube channels for this type of problem is “mindyourdecisions,” with Presh Talwalker. He also has some great books, such as “Mind Your Puzzles.”

If you wonder how teachers and text book authors create those challenging problems, It is quite simple. First, they determine the strategy that they want the student to practice. Then, they pick the answer and work backwards to create the problem. For example, I chose that the shirt should originally cost $29.99, the car was clocked at 200 mph, and worked backwards from there.