

Translation of An Object

Note: A diagonal translation can be done by performing a left/right translation and an up/down translation simultaneously.

Translation	Operation
Translation right by n.	$(x, y) \rightarrow (x+n, -y)$ Add n to all values of x.
Translation left by n.	$(x, y) \rightarrow (x-n, y)$ Subtract n from all values of x.
Translation up by n.	$(x, y) \rightarrow (x, y+n)$ Add n to all values of y.
Translation down by n.	$(x, y) \rightarrow (x, y-n)$ Subtract n from all values of y.
Translation left by n and up by m.	$(x, y) \rightarrow (x-n, y+m)$ Add the matrix $\begin{bmatrix} -n & . & . & -n \\ m & . & . & m \end{bmatrix}$

Reflection of An Object About An Axis

Note: The meaning of "reflection in x or y" is inconsistent in the texts. Make sure you know what your teacher means by this term.

Note: When multiplying matrices, order matters. The matrix below goes on the left and the coordinate matrix goes on the right. The number of columns in the left, reflection, matrix must match the number of rows in the right, coordinate, matrix.

Reflection	Operation
Reflection about the x-axis Reflection in y	$(x, y) \rightarrow (x, -y)$ Flip the sign of y. Multiply by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Reflection	Operation
Reflection about the y-axis Reflection in x	$(x, y) \rightarrow (-x, y)$ Flip the sign of x. Multiply by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection about the line $y = x$	Swap x and y, $(x,y) \rightarrow (y,x)$ Multiply by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Reflection about the line $y = -x$	Swap x and y and then flip signs of all x and y. $(x,y) \rightarrow (-y,-x)$ Multiply by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Rotating An Object About The Origin

Rotation	Operation
90 degrees counter clockwise or 270 degrees clockwise	$(x, y) \rightarrow (-y, x)$ Swap x and y. Flip sign of new x. Multiply by $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
180 degrees in either direction	$(x, y) \rightarrow (-x, -y)$ Flip the signs of x and y. Multiply by $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
270 degrees counter clockwise or 90 degrees clockwise	$(x, y) \rightarrow (y, -x)$ Swap x and y. Flip sign of new y. Multiply by $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Rotating An Object About a Point Other Than The Origin

Example: We will rotate the triangle A(1,1), B(5,5), C(-2,4) 90 degrees counter-clockwise about the point P(2,-1)

Step 1: Translate the triangle and the point of rotation 2 units (subtract 2 from every X) left and 1 unit up (add 1 to every Y) to place the point of rotation at the origin.

$$\begin{aligned}A (1,1) & \rightarrow A'(-1, 2) \\B (5,5) & \rightarrow B' (3, 6) \\C (-2,4) & \rightarrow C' (-4, 5)\end{aligned}$$

Step 2: Do the 90 degree counter-clockwise rotation using $(X,Y) \rightarrow (-Y,X)$

$$\begin{aligned}A (1,1) & \rightarrow A'(-1, 2) \rightarrow A''(-2, -1) \\B (5,5) & \rightarrow B' (3, 6) \rightarrow B''(-6, 3) \\C (-2,4) & \rightarrow C' (-4, 5) \rightarrow C''(-5, -4)\end{aligned}$$

Step 3: Translate the new triangle in the direction opposite to Step 1, 2 units (add 2 to every X) right and 1 unit down (subtract 1 from every Y).

$$\begin{aligned}A (1,1) & \rightarrow A'(-1, 2) \rightarrow A''(-2, -1) \rightarrow A''' (0, -2) \\B (5,5) & \rightarrow B' (3, 6) \rightarrow B''(-6, 3) \rightarrow B'''(-4, 2) \\C (-2,4) & \rightarrow C' (-4, 5) \rightarrow C''(-5, -4) \rightarrow C'''(-3, -5)\end{aligned}$$

